REAL NUMBERS

14-10-17.

Q1. If p is prime no., what is the LCM of p, p^2, p^3?
Ans. p^3

Q2. What is the HCF of $3^3 \times 5$ and $3^2 \times 5^2$?
Ans. 45

Q3. Examine $\frac{17}{30}$ is a terminating or not.
Ans. $\frac{17}{30} = \frac{17}{2 \times 3 \times 5}$ ∵ Denominator has 3 as its factor. So it is not terminating decimale.

Q4. Write a rational no. between $\sqrt{3}$ and $\sqrt{5}$.
Ans. $\boxed{\frac{9}{5}}$

Q5. Explain $3 \times 5 \times 7 + 7$ is a composite no.
Ans. $7(15+1) = 7 \times 16$, which has more than two factors.

Q6. What is the least no. that is divisible by all the numbers from 1 to 10?
Ans: Required no. = 2520

Q7. Find the sum $0.\overline{68} + 0.\overline{73}$
Ans. $\frac{68}{99} + \frac{73}{99} = \frac{141}{99} = \boxed{1.\overline{42}}$

Q8. What are possible values of remainder r, when a positive integer a is divided by 3?
Ans. $a = 3q + r$ where $0 \le r < 3$.

Q9. Write decimal expansion of $\frac{35}{70}$.
Ans. $\frac{35}{70} = \frac{35}{5^2 \times 2} = \frac{35 \times 2}{5^2 \times 2^2} = \frac{70}{10^2} = \boxed{0.70}$

16-10-17

Q10. Can two no. have 18 as their HCF and 380 as their LCM? Give Reason.

Ans. No, 18 does not divide 380.

Q11. Find the LCM and HCF of 12, 15, 21.

Ans.

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$HCF = 3$

$LCM = 2^2 \times 3 \times 5 \times 7 = 420.$

Q12. Can 4^n end with digit 0.

Ans. Prime factor of $4^n = (2 \cdot 2)^n$ Here 5 is not a factor so, it does not end with 0.

Q13. If HCF of a and b is 12 and product is 1800 what is LCM.

Ans. $1800 = 12 \times LCM$

$LCM = \frac{1800}{12}$

$\boxed{LCM = 150}$

Q14. Use Euclid's division Lemma find HCF of 960 and 432

Ans. $960 = 432 \times 2 + 96$

$432 = 96 \times 4 + 48$

$96 = 48 \times 2 + 0$

So $\boxed{HCF = 48}$

Q15. Write Euclid's division Lemma.

Ans. Given (two) integers a and b, ther

exist unique integers q and r satisfying
$a = bq + r$ where $0 \le r < b$.

Q16. Prove that $\sqrt{7}$ is an irrational no

Ans.

Let $\sqrt{7} = \frac{p}{q}$

$7 = \frac{p^2}{q^2}$

$p^2 = 7q^2$ — ①

7 divides $p^2 \Rightarrow$ 7 divides p

again, $p = 7m$

$p^2 = 49m^2$

from ① $49m^2 = 7q^2$

$q^2 = 7m^2$

7 divides $q^2 \Rightarrow$ 7 divides q.

It means 7 is a factor of p and q

So, our supposition is wrong.

$\therefore$ $\sqrt{7}$ is an irr. no.

Q17. show $5-\sqrt{3}$ is an irr. no.

Ans. Let $\frac{p}{q} = 5-\sqrt{3}$

$\sqrt{3} = \frac{5q-p}{q}$

RHS is ~~irr no.~~ rational no and LHS is irr. no.

So it is not possible.

$\therefore$ $5-\sqrt{3}$ is an irr. no.

Q18. If $\frac{p}{q}$ is a rational no. what is the cond[ition] on q so that the decimal expansion [of] $\frac{p}{q}$ is terminating?

Ans. The prime factorisation of q should be of the form $2^m 5^n$.

Q19. State the fundamental theorem of arithmetic.

Ans. Every composite number ca[n] be expressed as a product [of] primes and this expression is unique.

Q20 Use Euclid's division lemma to sh[ow] that the square of any (+ve) integer i[s] either of the form $3m$ or $3m+1$.

Ans. Let $a = 3q + r$ $r = 0, 1, 2,$

when $r = 0$

$$a = 3q + 0$$

$$a^2 = 9q^2 = 3 \cdot 3q^2 = 3m$$

$r = 1 \Rightarrow$ $a = 3q + 1$

$$a^2 = 9q^2 + 6q + 1$$

$$= 3(3q^2 + 2q) + 1$$

$$= 3m + 1$$

$r = 2 \Rightarrow$ $a = 3q + 2$

$$a^2 = 9q^2 + 12q + 4$$

$$= 3(3q^2 + 4q + 1) + 1$$

$$= 3m + 1$$

Linear Eq^n in two variables.

Q1. Find the coordinate where the line $x-y=8$ will intersect y-axis.

Ans. x should be 0.

$\therefore 0-y=8,\ y=-8$

Required coordinate $(0,-8)$.

Q2. Write the condition for coincident lines.

Ans. $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Q3. Is $2ax+by=a$ and $4ax+2by-2a=0$ consistent.

$\frac{a_1}{a_2}=\frac{2a}{4a}=\frac{1}{2}$

$\frac{b_1}{b_2}=\frac{b}{2b}=\frac{1}{2}$ $\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$

yes it is consistent.

$\frac{c_1}{c_2}=\frac{-a}{-2a}=\frac{1}{2}$

Q4. Find the value of k for which $kx-y=2$, $6x-2y=3$ has unique sol^n?

Ans. For unique sol^n.

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\Rightarrow\frac{k}{6_3}\neq\frac{-1}{-2}\Rightarrow\boxed{k\neq 3}$

Q5. Solve: $152x-378y=-74$ —(i)

$-378x+152y=-604$ —(ii)

(i) + (ii) $-226x-226y=-678$

$x+y=3$ —(iii)

(i) − (ii) $530x-530y=530$

$x-y=1$ —(iv)

(iii) + (iv)

$x+\cancel{y}=3$

$x-\cancel{y}=1$

$2x=4$

$\boxed{x=2}$

$2+y=3\Rightarrow\boxed{y=1}$

Q6. solve for x and y:

$$\frac{b}{a}x + \frac{a}{b}y = a^2+b^2 \text{ —(i)} \quad x+y = 2ab \text{ —(ii)}$$

$$\text{(ii)} \times \frac{b}{a} \Rightarrow \frac{b}{a}x + \frac{b}{a}y = 2b^2 \text{ —(iii)}$$

$$\text{(i)}-\text{(iii)} \quad \left(\frac{a}{b}-\frac{b}{a}\right)y = a^2+b^2-2b^2$$

$$y = \frac{\cancel{a^2-b^2}}{\cancel{a^2-b^2}} \times ab$$

$$y = ab$$

from (ii)

$$x+y = 2ab$$

$$x+ab = 2ab$$

$$\boxed{\begin{aligned} x &= ab \\ y &= ab \end{aligned}}$$

Q7. solve for x and y:

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad \text{and} \quad \frac{5}{x-1} + \frac{1}{y-2} = 2$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

$$6p - 3q = 1 \quad \text{and} \quad 5p + q = 2$$

$$q = 2-5p$$

~~6(2-5p)~~

$$6p - 3(2-5p) = 1$$

$$6p - 6 + 15p = 1$$

$$21p = 7$$

$$p = \frac{1}{3}$$

$$q = 2 - \frac{5}{3} = \frac{1}{3}$$

$$x-1 = 3 \Rightarrow y-2 = 3$$

$$\boxed{x = 4, \quad y = 5}$$

Q8. In $\triangle ABC$, $\angle A = x$, $\angle B = 3x$, $\angle C = y$ if $3y - 5x = 30$
Show it is right angled $\triangle$.

Sol^n:

$\angle A + \angle B + \angle C = 180^\circ$

$x + 3x + y = 180$

$4x + y = 180$ — (i)

$3y - 5x = 30$ — (ii)

after solving we have $x = 30^\circ$

$\angle A = 30$, $\angle B = 90^\circ$, $\angle C = 60^\circ$

So, it is right angled $\triangle$.

Q9. Show $2x - 3y = 5$, $6x - 9y = 15$ has infinite no. of solutions.

Sol^n. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ for infinite no. of sol^{ns}.

$\frac{2}{6} = \frac{-3}{-9} = \frac{5}{15}$

$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$

Q10. Find K if $Kx - y = 2$, $6x - 2y = 3$ has no solutions.

Ans. For no. sol^n:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$= \frac{K}{6} = \frac{-1}{-2} \neq \frac{2}{3}$

$\boxed{K = 3}$ & $K \neq 4$

Ans.

Q11. Solve for x & y: $\sqrt{2}x - \sqrt{3}y = 0$ & $\sqrt{5}x + \sqrt{2}y = 0$

Ans. $\sqrt{2}x - \sqrt{3}y = 0$ — (i)

$\sqrt{5}x + \sqrt{2}y = 0$ — (ii)

(i) $\times \sqrt{2}$ + (ii) $\times \sqrt{3}$ we get, $(2 + \sqrt{15})x = 0 \Rightarrow \boxed{x = 0}$

then, $\sqrt{2} \times 0 - \sqrt{3}y = 0 \Rightarrow \boxed{y = 0}$. Ans

Q12. Solve: $\frac{1}{2x} - \frac{1}{y} = 1$ and $\frac{1}{x} + \frac{1}{2y} = 8$

Ans.

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$

So. $\frac{u}{2} - v = 1 \Rightarrow u - 2v = 2$ — (i)

and $u + \frac{v}{2} = 8 \Rightarrow 2u + v = 16$ — (ii)

(ii) × 2 + i we get $5u = 30$

$u = 6$

Put $u = 6$ in (i) $6 - 2v = -2$

$-2v = -8$

$v = 4$

again $\frac{1}{x} = 6 \Rightarrow \boxed{x = \frac{1}{6}}$

$\frac{1}{y} = 4 \Rightarrow \boxed{y = \frac{1}{4}}$

Q13. Solve: $6x + 3y = 7xy$

$3x + 9y = 11xy$

Ans.

Dividing both eq^n by xy.

$\frac{6}{y} + \frac{3}{x} = 7$ — (i) $\frac{3}{y} + \frac{9}{x} = 11$ — (ii)

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$

So, $3u + 6v = 7$ — (iii) and $9u + 3v = 11$ — (iv)

(iv) × 2 − (iii) we get $15u = 15 \Rightarrow u = 1$

Put $u = 1$ in (iii) we get $6v = 4$

$v = \frac{2}{3}$

Now $\frac{1}{x} = 1$ & $\frac{1}{y} = \frac{2}{3}$

$\boxed{x = 1}$ $\boxed{y = \frac{3}{2}}$

Q14. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the n^r and it becomes $\frac{1}{4}$ when 8 is added to its D^r. Find fraction

Soln. Let $N^r = x$, $D^r = y$

A/Q $\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x-3 = y$ — (I)

$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x = y+8$

$4x - y = 8$ — (II)

from (I) $4x - 3x + 3 = 8$

$x = 5$

Put $x = 5$ in (I) $3\times5-3 = y$

$y = 12$

So required fraction $= \boxed{\frac{5}{12}}$ Ans.

Q15. Five years ago A was thrice as old as B and 10 years later A shall be twice as old as B. What are present ages?

Soln: Let present age of A $= x$

" " B $= y$.

A/Q $y-5 = 3(x-5) \Rightarrow 3x - y = 10$ — (I)

again A/Q $y+10 = 2(x+10) \Rightarrow 2x - y = -10$ — (II)

(II) − (I) we get $x = 20$

Put in (I) $y = 50$

A's Age = 20 years

B's Age = 50 years] Ans

Q16. The sum of two no^s is 8 and the s of their reciprocals is $\frac{8}{15}$. Find no^s.

Sol^n: Let no be x & y.

Case 1. $x+y=8$

Case II $\Rightarrow$ $\frac{1}{x}+\frac{1}{y}=\frac{8}{15} \Rightarrow \frac{x+y}{xy}=\frac{8}{15}$

$\frac{8}{xy}=\frac{8}{15} \Rightarrow xy=15$

$x-y=\sqrt{(x+y)^2-4xy}$

$=\sqrt{8^2-4\times15}$

$=\sqrt{4}$

$x-y=\pm2$

$x+y=8$	$x+y=8$
$x-y=2$	$x-y=-2$
$x=5, y=3$	$x=3$, $y=5$

$\boxed{\text{required no} = 3 \,\&\, 5}$ Ans

Q17. Sum of two no^s is 35 and their difference is 13. Find the no^s.

Soln: Let two no^s are x and y

A/Q $x+y=35$ — (I) and $x-y=13$ (II)

(I) + (II) we get $2x=48 \Rightarrow \boxed{x=24}$

Put in (I) $24+y=35 \Rightarrow \boxed{y=11}$ Ans

Q18. A person buys a horse and a cart for Rs. 1200. He sells the horse at 12% profit and cart at 6% loss. If he makes a profit at 6% on whole. Find the cost of the horse.

Soln:

Let cost of horse be Rs x & cart be Rs. y

using information $x+y=1200$ —(I)

and $\frac{112x}{100}+\frac{94}{100}y=\frac{106}{100}\times 1200$

$112x+94y=127200$

$56x+47y=63600$ —(II)

(I)×47 − (II) we get $9x=7200$

$\boxed{x=800}$ Ans.

Q19. A person can row downstream 20 km in 2 hrs. and upstreams 4 km in 2 hrs. Find man's speed of rowing in still water and the speed of current.

Soln Let man's speed $= x$ km/hr

Speed of current $= y$ km/hr

A/Q $\frac{20}{x+y}=2 \Rightarrow x+y=10$ —(I)

and $\frac{4}{x-y}=2 \Rightarrow x-y=2$ —(II)

(I)+(II) we get $2x=12 \Rightarrow \boxed{x=6}$

$x=6$ Put in (I) $\Rightarrow y=10-6 \Rightarrow \boxed{y=4}$ As.

Q20. How many solns have eqn: $y=0$ & $y=-7$.

soln. ∵ they are parallel to each other so they have no solns.