– Surface Area & Volumes –

(1) 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

(2) A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find total surface area of the toy.

(3) A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

(4) A wooden article was made by scooping out a cylinder, as shown in fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find total surface area of the article.

(5) A vessel is in the form of inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

(6) Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively, are melted to form a single solid sphere. Find the radius of resulting sphere.

(7) A well of diameter 3 m dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find height of embankment

(8) How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm.

(9) Water Canal 6 m wide & 1.5 m deep is flowing with Speed of 10 km/hr. How much area will it irrigate in 30 min. if 8 cm of standing water is needed.

(10) A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical Tank in his field which is 10 m diameter and 2 m deep. If water flows through the Pipe at the rate of 3 km/hr in how much time will the tank filled.

(11) The slant height of a frustum of a cone is 4 cm and Perimeters of its circular ends are 18 cm and 6 cm. find curved S.A. of frustum.

(12) A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find capicity of the glass.

(13) A metallic right circular cone 20 cm high and whose vertical Angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm. Find length of wire.

(14) The S.A. of a sphere is 616 cm find its Radius.

(15) A cylinder and a cone are of the same base radius and same height. Find Ratio of volume of cylinder to cone.

(16) The slant height of the frustum of cone is 5 cm. difference of Radii is 4 cm, Write the height of the frustum.

(17) The slant height of a frustum of a cone is 10 cm. height is 8 cm then find difference of Radii.

(18) A solid sphere of Radius 10.5 cm is melted and Recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. Find No of cones.

(19) A bucket is in the form of a frustum of a cone holds 28.49 l milk and Radii are 28 cm & 21 cm. Find height of bucket.

(20) A solid right circular cone of diameter 14 cm and height 4 cm is melted to form hollow hemisphere. If the external diameter is 10 cm, Find internal diameter.

Sol^{n}

(1) $\because$ Volume of cube $= 64\,cm^3$

$a^3 = 64$

$a = \sqrt[3]{64} = 4\,cm.$

length of New cubaid $= 4+4 = 8\,cm.$

breadth " " $= 4\,cm.$

height " " $= 4\,cm.$

S.A. of New cubaid $= 2(lb + bh + hl)$

$= 2(8\times4 + 4\times4 + 4\times8)\,cm^2$

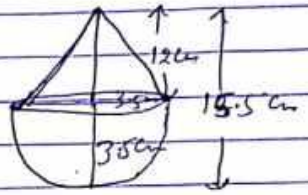
$= 2(32+16+32)$

$= 2 \times 80 = 160\,cm^2$ Ans

(2) Radius $= 3.5\,cm$

height of cone $= 15.5 - 12.5$

$= 12.0\,cm.$



Slant height of cone

$l = \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + (3.5)^2}$

$= \sqrt{144 + 12.25}$

$= \sqrt{156.25} = 12.5\,cm$

Total S.A. of toy = S.A. of hemisphere + S.A. of cone

$= 2\pi r^2 + \pi r l$

$= \pi r (2r + l)$

$= \frac{22}{7} \times 3.5\,(2\times3.5 + 12.5)$

$= 11.0 \times (7 + 12.5)$

$= 11 \times 19.5$

$= 214.5\,cm^2$ Ans

(3) edge of cube $= l$

S.A. of cube $= 6l^2$

Radius of hemisphere $= \frac{l}{2}$

Total S.A. of Solid = S.A. of Cube

+ S.A. of hemisphere − A. of circle.

$= 6l^2 + 2\pi r^2 + \pi r^2$

$= 6l^2 + \pi r^2$

$= 6l^2 + \pi \left(\frac{l}{4}\right)^2$

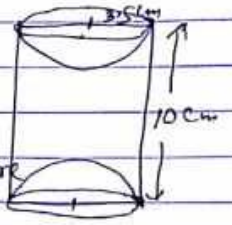
$= 6l^2 + \pi \frac{l^2}{4}$

$= \frac{24l^2 + \pi l^2}{4}$

$= \frac{l^2}{4}(24 + \pi)\ cm^2$ Ans

(4) height of cylinder = 10 cm.

radius = 3.5 cm.



Total S.A. = C.S.A. of cylinder + 2 C.S.A of hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 (10 + 2 \times 3.5)$

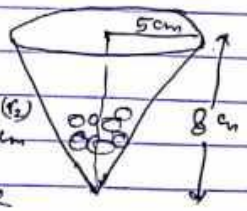
$= 22 \times (10 + 7)$

$= 22 \times 17$

$= 374\ cm^2$ Ans

(5) height of cone = 8 cm.

Radius of cone (r_2) = 5 cm.



Let No of Lead shots = x

Radius of lead shots (r_1) = 0.5 cm

A/Q x × V. of Lead shots = $\frac{1}{4}$ × V. of cone

$x \times \frac{4}{3}\pi r_1^3 = \frac{1}{4} \times \frac{1}{3}\pi r_2^2 h$

$x = \frac{1}{4} \times \frac{h \times r_2^2}{4 \times r_1^3}$

$= \frac{8 \times (5)^2}{16 \times (0.5)^3} = \frac{8 \times 25}{2 \cdot 16 \times 0.125}$

$$= \frac{25000}{2 \times 125}$$

$$= 100 \text{ lead shots}$$

Ans

(6) $r_1 = 6\text{ cm}$

$r_2 = 8\text{ cm}$

$r_3 = 10\text{ cm}$.

Let Radius of New sphere $= r$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_1^3 + r_2^3 + r_3^3)$$

$$r^3 = (6^3 + 8^3 + 10^3)$$

$$= 216 + 512 + 1000$$

$$r^3 = 1728$$

$$r = \sqrt[3]{1728} = 12\text{ cm}$$

Ans

(7) Diameter of well $= 3\text{ m}$

Radius $= \frac{3}{2}\text{ m}$.

Depth of well $h_1 = 14\text{ m}$

Let height of embankment $= h$.

Inner Radius of embankment $(r_1) = \frac{3}{2}\text{ m} = 1.5\text{ m}$

Outer " " $(r_2) = \frac{3}{2} + 4$

$= 1.5 + 4$

$= 5.5\text{ m}$

$\therefore$ V. of embankment $=$ V. of earth dug

$$\pi h (r_2^2 - r_1^2) = \pi r^2 h_1$$

$$h\left[(5.5)^2 - (1.5)^2\right] = \left(\frac{3}{2}\right)^2 \times 14$$

$$h\,[5.5 + 1.5][5.5 - 1.5] = \frac{9}{4_2} \times 14 = \frac{63}{2}$$

$$h \times 7 \times 4 = \frac{63}{2}$$

$$h = \frac{63}{2 \times 28}$$

$$= \frac{63}{56}$$

$$= 1.125 \text{ m}$$

Ans

(8) Diameter of coin = 1.75 cm

Radius = $\frac{1.75}{2}$ cm.

Thickness = 2 mm = 0.2 cm.

Let No. of coins = n.

A/Q.

$n \times$ V. of coin = V. of cuboid

$$n \times \pi r^2 h = l \times b \times h$$

$$n \times \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times 0.2 = 5.5 \times 10 \times 3.5$$

$$n = \frac{55 \times 35 \times 2}{22 \times 0.25 \times 1.75 \times 0.1}$$

$$= \frac{55 \times 35 \times 1000 \times 2}{22 \times 25 \times 175 \times 1}$$

$$= \frac{10000 \times 0.1 \times 2}{1 \times 5}$$

$$= 4000$$

$$= 400 \text{ coins.}$$ Ans

(9) Width of canal (b) = 6 m

Depth (h) = 1.5 m.

Rate of flowing water = 10 km/hr

length of water in 30 min (l) $= \frac{10 \times 1000}{60} \times 30$

$= 5000$ m

Height of water in field = 8 cm

$= \frac{8}{100}$ m

$= 0.08$ m

∴ V. of water in field = V. of water in canal

$l \times b \times h = l_1 \times b_1 \times h_1$

$\text{Area} \times 0.08 = 5000 \times 6 \times 1.5$

$\text{Area} = \frac{5000 \times 9}{0.08}$

$= \frac{5625000 \times 9}{8}$

$= 5625000\ m^2$

(10) Diameter of pipe = 20 cm

radius = 10 cm.

Rate of flowing = 3 km/hr.

Let time taken = t hours.

Volume of water in t hours

$= \pi \left(\frac{10}{100}\right)^2 \times 3 \times 1000 \times t$

$= \pi \frac{1}{100} \times 3000 \times t$

$= 30\pi t\ m^3$

V. of water in cylindrical tank $= \pi r^2 h$

$= \pi (5)^2 \times 2$

A/Q.

$30 \times \pi \times t = \pi \times 25 \times 2$

$t = \frac{25 \times 2}{30} = \frac{5}{3}$ hours

Ans

(11) Slant height (l) = 4 cm

Upper circumference = 18 cm

$2\pi r_1 = 18$ cm

$\pi r_1 = \frac{18}{2} = 9$ cm.

Lower circumference = 6 cm

$2\pi r_2 = 6$

$\pi r_2 = \frac{6}{2}$ 3

$\pi r_2 = 3$ cm.

C.S.A. of frustum $= \pi(r_1 + r_2)l$

$= (\pi r_1 + \pi r_2) \times l$

$= (9 + 3) \times 4$

$= 12 \times 4$

$= 48$ cm^2

(12) Upper diameter = 4 cm.

" Radius (r_1) = 2 cm.

Lower diameter = 2 cm

" Radius (r_2) = 1 cm.

Height of glass (h) = 14 cm.

Capacity of glass = V. of frustum

$= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{22}{7} \times \frac{14}{3}(2^2 + 1^2 + 2 \times 1)$

$= \frac{44}{3} \times (4 + 1 + 2)$

$= \frac{44 \times 7}{3}$

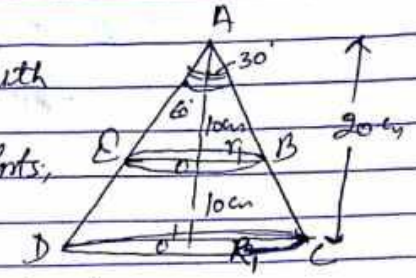
$= \frac{308}{3}$ cm^3

Ans

(13)

Let ADC is a cone with vertical Angle 60°

Cone is cut into two parts, ∥ to its ~~height~~ base at height 10 cm.



Radius of larger end = R_1

In $\triangle AO'C$ $\tan 30° = \frac{O'C}{O'A}$

$$\frac{1}{\sqrt{3}} = \frac{R_1}{20}$$

$$R_1 = \frac{20}{\sqrt{3}} \text{ cm}$$

In $\triangle AOB$ $\tan 30° = \frac{r_1}{OA}$

$$\frac{1}{\sqrt{3}} = \frac{r_1}{10}$$

$$r_1 = \frac{10}{\sqrt{3}} \text{ cm}$$

$$\text{V. of frustum} = \frac{\pi h}{3}(R_1^2 + r_1^2 + R_1 r_1)$$

$$= \pi \frac{10}{3}\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right]$$

$$= \pi \frac{10}{3}\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right]$$

$$= \pi \frac{10}{3} \times \frac{700}{3}$$

∵ Let length of wire = h

∵ Volume of wire = V. of frustum

$$\pi r^2 h = \pi \frac{10}{3} \times \frac{700}{3}$$

$$\frac{1}{32} \times \frac{1}{32} \times h = \frac{10}{3} \times \frac{700}{3}$$

$$h = \frac{7000}{9} \times 32 \times 32$$

$$= \frac{7000 \times 924}{9}$$

$$= \frac{6468000}{9}$$

$$= 7964.4 \text{ m}$$

Ans

(14) S.A. of sphere = 616 cm²

$4\pi r^2 = 616$

$4 \times \frac{22}{7} \times r^2 = 616$

$r^2 = \frac{616 \times 7}{4 \times 22}$

$r^2 = 49$

$r = \sqrt{49}$

$= 7$ cm. Ans

(15) Let Radius of cylinder and cone $= r$

height " " " $= h$

Ratio of Volumes $= \frac{\text{V. of cylinder}}{\text{V. of cone}}$

$= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h}$

$= \frac{3}{1}$

$= 3:1$ Ans

(16) Slant height of frustum $(l) = 5$ cm.

diff. of Radii $(r_1 - r_2) = 4$ cm.

$\because l = \sqrt{h^2 + (r_1 - r_2)^2}$

$l^2 = h^2 + (r_1 - r_2)^2$

$5^2 = h^2 + 4^2$

$25 - 16 = h^2$

$h^2 = 9$

$h = \sqrt{9} = 3$ cm Ans

(17) Slant height (l) = 10 cm
height (h) = 8 cm.

$\therefore l = \sqrt{h^2 + (r_1 - r_2)^2}$.

$l^2 = h^2 + (r_1 - r_2)^2$

$10^2 = 8^2 + (r_1 - r_2)^2$

$100 - 64 = (r_1 - r_2)^2$

$36 = (r_1 - r_2)^2$

$r_1 - r_2 = \sqrt{36}$

$= 6$ cm

diff. of Radii = 6 cm. Ans

(18) Radius of sphere (r_1) = 10.5 cm.
Radius of cone (r_2) = 3.5 cm.
height cone = 3 cm.

$\Rightarrow$ Let No of cones = x

$x \times$ V. of cone = V. of sphere

$x \times \frac{1}{3} \pi r_2^2 h = \frac{4}{3} \pi r_1^3$

$x \times \frac{1}{3} \times (3.5)^2 \times 3 = \frac{4}{3} \times (10.5)^3$

$x = \frac{4 \times 10.5 \times 10.5 \times 10.5 \times 3}{3 \times 3.5 \times 3.5}$

$= 42.0 \times 3$

$= 126$ cones

Ans

(19) Capicity of frustum = 28.49 l

$\therefore V = \frac{28.49}{1000} m^3$

$\because R = 28cm = 0.28m$

$r = 21cm = 0.21 m$

$\because V = \frac{1}{3}\pi h [R^2 + r^2 + R.r]$

$\frac{28.49}{1000} = \frac{1}{3} \times \frac{22}{7} \times h [(0.28)^2 + (0.21)^2 + 0.28 \times 0.21]$

$\frac{2849}{100000} = \frac{22}{21} \times h \left[\frac{28}{100} \times \frac{28}{100} + \frac{21}{100} \times \frac{21}{100} + \frac{28}{100} \times \frac{21}{100}\right]$

$\frac{2849}{100000} = \frac{22}{21} \times h \left[\frac{784 + 441 + 588}{10000}\right]$

$\frac{2849}{100000} = \frac{22}{21} \times h \left[\frac{1813}{10000}\right]$

$h = \frac{2849 \times 21 \times 3}{22 \times 1813 \times 10}$

$= \frac{3}{20} = 0.15 m$

$= 15cm$ Ans

(20) diameter of Cone = 14 cm.

Radius (r_1) = 7 cm

height (h) = 8 cm.

External diameter of sphere = 10 cm.

External Radius (r_2) = 5 cm.

Let internal Radius = R.

$\therefore$ V. of sphere = V. of Cone

$\frac{4}{3}\pi (r^3 - R^3) = \frac{1}{3}\pi r_1^2 h$

$4 (5^3 - R^3) = 7 \times 7 \times 8$

$125 - R^3 = \frac{7 \times 7 \times 8}{4}$

$R^3 = 125 - 98$

$= 27$

$R = \sqrt[3]{27} = 3cm.$

Internal diameter

$= 2r$

$= 2 \times 3$

$= 6 cm$ Ans