Probability

Question:-1. A can solve 90% of the Problems given in a book. and B can solve 70%. What is the Probability that at least one of them will solve a Problem selected at random from the book.?

Sol:- Let E_1 = event that A solves the Problem and E_2 = event that B solve the Problem. Then $P(E_1) = \frac{90}{100} = \frac{9}{10}$ and $P(E_2) = \frac{70}{100} = \frac{7}{10}$

Clearly E_1 and E_2 are indepdent event
$P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{9}{10} \times \frac{7}{10} = \frac{63}{100}$

P (at least one of them will solve the Problem) $= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= \left(\frac{9}{10} + \frac{7}{10} - \frac{63}{100}\right) = \left(\frac{90+70-63}{100}\right) = \frac{97}{100}$

Hence the required Probability is 0.97

Question 2. In a ~~bd~~ bolt factory three machines A, B, C manufacture 25% 35% and 40% of the total Production respectively. of their respective outputs 5%, 4% and 2% are defective. A bolt is drawn at random from the total Product and it is found to be defective. find the Probability that it was manufactured by the machine C.

Sol: 2. Let E_1, E_2 and E_3 be the events of drawing a bolt Produced by Machine A, B, and C respectively Then

$P(E_1)=\frac{25}{100}=\frac{1}{4}$, $P(E_2)=\frac{35}{100}=\frac{7}{20}$, $P(E_3)=\frac{40}{100}$

Let E be the event of drawing a defective bolt Then

$P(E/E_1)=\frac{5}{100}$, $P(E/E_2)=\frac{4}{100}$

$P(E/E_3)=\frac{2}{100}=\frac{1}{50}$

Probability that the bolt drawn is manufactured by C given that it is defective. $P(E_3/E)$.

$$= \frac{P(E/E_3)\cdot P(E_3)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)+P(E/E_3)P(E_3)}.$$

$$= \frac{\frac{1}{50}\times\frac{2}{5}}{\frac{1}{20}\times\frac{1}{4}+\frac{1}{25}\times\frac{7}{20}+\frac{1}{50}\times\frac{2}{5}} = \frac{1}{125}\times\frac{2000}{69}$$

$$= \frac{16}{69}$$

Hence the Required Probability is $\frac{16}{69}$.

Q3. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Sol: Clearly, the sample space is $S=\{1,2,3,4,5,6,7,8,9,10\}$

Let A = Event that the number on the drawn card is even, and

B = Event that the number on the drawn card is more than 3.

then $A = \{2, 4, 6, 8, 10\}$

$B = \{4, 5, 6, 7, 8, 9, 10\}$

and $A \cap B = \{4, 6, 8, 10\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{10} \text{ and}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

Suppose B has already occured and then A occurs.

So, we have to find $P(A/B)$

Now $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/5}{7/10}$

$$= \frac{2}{5} \times \frac{10}{7} = \frac{4}{7}$$

Hence the required probability is $\frac{4}{7}$.

Q4. A card from the pack of 52 card is lost. From the remaining card of the pack, two cards are drawn and are found to be both spades. Find probability of the lost card being a spade.

Sol: Let E_1, E_2, E_3 and E_4 be the event of losing a card of spades, cubes, Hearts and diamonds respectively

Then $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$

Let E be the event of drawing 2 spades. from the remaining 51 cards, then,

$P(E/E_1)$ = probability of drawing 2 spades, given That the a card of spades is missing

$$= \frac{12C_2}{51C_2} = \frac{12\times 11}{2} \times \frac{2}{(51\times 50)} = \frac{22}{425}$$

$P(E/E_2)$ = probability of drawing 2 spades, given That the cards of cubes is missing

$$= \frac{13C_2}{51C_2} = \frac{13\times 12}{2} \times \frac{2}{51\times 50}$$

$$= \frac{26}{425}$$

$P(E/E_3)$ = probability of drawing 2 spades, given that a card of heart is missing

$$= \frac{13C_2}{51C_2} = \frac{26}{425}$$

$P(E/E_4)$ = probability of drawing 2 spades, given that a card of diamonds. is missing

$$= \frac{13C_2}{51C_2} = \frac{26}{425}$$

$P(E_1/E)$ = probability of the lost card being spade, given that 2 spades are drawn from the remaining

$$= \frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\cdot P(E/E_2) + P(E_3)\cdot P(E/E_3) + P(E_4)\cdot P(E/E_4)}$$

$$= \frac{\left(\frac{1}{4}\times 22/425\right)}{\left(\frac{1}{4}\times\frac{22}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)+\left(\frac{1}{4}\times\frac{26}{425}\right)}$$

$$= \frac{22}{100} = 0.22.$$

Hence Required probability is 0.22.

Q5. Two Cards are drawn Simultaneously (or Successively without replacement) from a well shuffled pack of 52 Cards. Find the mean and Variance of the number of aces.

Sol: Let x be the random Variable.
Then x, denotes the number of aces in a draw of two cards.

$\therefore$ x can assume the value 0, 1 or 2
Number of ways of drawing 2 cards out of 52 = $C(52, 2)$

$$P(x=0) = P(\text{both non aces})$$
$$= \frac{48C_2}{52C_2} = \frac{188}{221}$$

$P(x=1)$ P(one ace out of 4) and (one non ace out of 48)
$$= \frac{4C_1 \times 48C_1}{52C_2} = \left(\frac{4\times 48}{52\times 51}\times 2\right)$$
$$= \frac{32}{221}$$

$P(X=2) = P(\text{both aces})$
$= {}^4C_2 / {}^{52}C_2$
$= \left(\frac{4\times3}{2\times1} \times \frac{2\times1}{52\times51}\right) = \frac{1}{221}$

Thus, we have the following probability distribution

$X = x_i$	0	1	2
p_i	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean $\mu = \sum x_i p_i$
$= \left(0\times\frac{188}{221}\right) + \left(1\times\frac{32}{221}\right) + \left(2\times\frac{1}{221}\right)$
$= 2/13$

Variance $\sigma^2 = \sum x_i^2 p_i - \mu^2$
$= \left(0\times\frac{188}{21}\right) + \left(1\times\frac{32}{221}\right) + \left(4\times\frac{1}{221}\right) - \frac{4}{169}$
$= \left(\frac{36}{221} - \frac{4}{169}\right)$
$= \frac{400}{2873}.$

Q6. An instructor has a question bank consisting of 300 easy true/False questions 200 difficult true/false questions 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Sol. 6, Clearly the sample space consists of 1400 questions

$n(S) = 1400$

Let A = event of selecting an easy question and

B = event of selecting a multiple choice question

Then $A \cap B$ = event of selecting an easy multiple choice question

$n(A) = (300+500) = 800$, $n(B) = 500+400 = 900$

and $n(A \cap B) = 500$

$P(A) = \frac{n(A)}{n(S)} = \frac{800}{1400} = \frac{4}{7}$, $P(B) = \frac{n(B)}{n(S)} = \frac{900}{1400}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{500}{1400} = \frac{5}{14}$

Suppose B has already occurred and then A occurs.

Thus we have to find $P(A/B)$.

Now $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{5/14}{9/14} = 5/9$

Hence required probability is $5/9$.

Q.7. Find the binomial distribution for which the mean and variance are 12 and 3 respectively.

Solution

Let X be a binomial variate for which mean = 12 and variance = 3

Then $np = 12$, $npq = 3$, $q = \frac{1}{4}$

$p = (1-q) = (1-\frac{1}{4}) = 3/4$

and $np = 12$, $n \times \frac{3}{4} = 12 \Rightarrow n = 16$

$n = 16$, $p = 3/4$, $q = \frac{1}{4}$

Hence the binomial distribution is given by

$$P(X=r) = {}^{16}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{16-r} \text{ where } r = 0, 1, 2, \ldots 15.$$

Question 8.

In a binomial distribution Prove that mean > Variance.

Solution: Let X be a binomial Variate with parameters n and p. Then

Mean $= np$, and Variance $= npq$

Mean − Variance $= np - npq = np(1-q)$

$npp = np^2 > 0$

(Mean − Variance) > 0

Mean > Variance

Hence Mean > Variance.