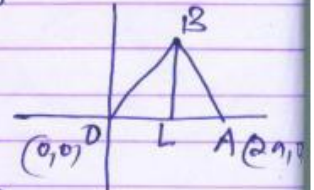
COORDINATE GEOMETRY

Q1. Write the coordinates of ~~x and~~ point which lies on x-axis and y-axis.

Ans. On x-axis = $(x, 0)$
On y-axis = $(0, y)$

Q2. Find the coordinates of a point (vertex) of an equilateral triangle of side 2a

Soln: $OA = OB = AB = 2a$
Let $BL \perp OA$
$OL = LA = a$



In $\triangle OLB$,

$$OB^2 = OL^2 + LB^2$$
$$(2a)^2 = a^2 + LB^2$$
$$LB^2 = 3a^2$$
$$\boxed{LB = \sqrt{3}a}$$

coordinates of $O = (0,0)$, $A = (2a, 0)$, $B = (a,$

Q3. Find the distance between $(-6, 7)$ and $(-1, -$
Soln: $x_1 = -6$, $y_1 = 7$, $x_2 = -1$, $y_2 = -5$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1+6)^2 + (-5-7)^2}$$
$$= \sqrt{5^2 + (-12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$\boxed{AB = 13}$$ Ans

Q4. Find x if distance between points $(x,-1)$ and $(3,2)$ is 5.

Sol^n: Let $P(x,-1)$ and $Q(3,2)$

$PQ = 5$

$PQ^2 = 25$

$(x-3)^2 + (2+1)^2 = 25$

$x^2+9-6x+9-25=0$

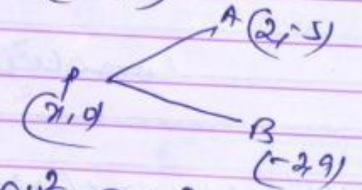
$x^2-6x-7=0$

$(x-7)(x+1)=0$

$\boxed{x=7 \text{ or } x=-1}$ Ans.

Q5. Find a point on x-axis which is equidistant from $A(2,-5)$ and $B(-2,9)$.

Sol^n:



ATQ $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2+(0+5)^2 = (x+2)^2+(0-9)^2$

$\Rightarrow x^2+4-4x+25 = x^2+4+4x+81$

$-8x = 81-25$

$x = \frac{56}{-8}$

$x=-7$

$\boxed{\text{So Point} = (-7,0)}$ Ans.

Q6. The x- coordinate of a point P is twi[ce] its y- coordinate. If P is equidist[ant] from Q (2, -5) and R (-3, 6), then f[ind] the coordinates of P.

Sol^n: Let coordinates of P be (x, y).
It is given that $x = 2y$.

$PQ = QR \Rightarrow PQ^2 = QR^2$

$(x-2)^2 + (y+5)^2 = (x+3)^2 + (y-6)^2$

$(2y-2)^2 + (y+5)^2 = (2y+3)^2 + (y-6)^2$

$\cancel{5y^2} + 2y + 29 = \cancel{5y^2} - 0y + 45$

$2y = 45 - 29$

$2y = 16$

$y = 8$

coordinates of P = (16, 8) Ans

Q7. Show that the points (1, -1) (5, 2) and (9[, 5)] are collinear.

Sol^n. Let $A = (-1, 1)$, $B = (5, 2)$, $C = (9, 5)$

$AB = \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16+9} = 5$

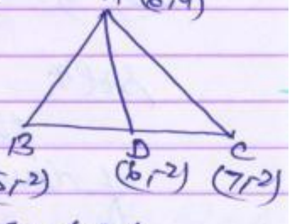
$BC = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{16+9} = 5$

$AC = \sqrt{(1-9)^2 + (-1-5)^2} = \sqrt{64+36} = 10$

$AC = AB + BC$

Hence, A, B, C are collinear. An[s]

Q8. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle. Also find the length of median through A.



Soln: $AB = \sqrt{(6-5)^2+(4+2)^2} = \sqrt{37}$

$AC = \sqrt{(6-7)^2+(4+2)^2} = \sqrt{37}$

$AB = AC$ so ABC is isosceles Δ.

$\because$ AD is median $\therefore$ D is the mid point of BC.

coordinates of $D = \frac{5+7}{2}, \frac{-2-2}{2}$ i.e. $(6,-2)$

$\therefore AD = \sqrt{(6-6)^2+(4+2)^2} = 6$ Ans.

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9. Find the coordinates of point which divides the line joining the points $(6,3)$ and $(-4,5)$ in the ratio 3:2 internally.

Soln: $x_1 = 6$, $y_1 = 3$ $\quad x_2 = -4$, $y_2 = 5$ $\quad m_1 = 3$, $m_2 = 2$ $\quad (6,3)$ P (x,y) $(-4,5)$

$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$, $\quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$x = \frac{3 \times -4 + 2 \times 6}{3+2}$, $\quad y = \frac{3 \times 5 + 2 \times 3}{3+2}$

$x = 0$, $\quad y = \frac{21}{2}$

So Point $P = (0, \frac{21}{2})$ Ans.

Q10. In what ratio does the x-axis divi the line segment joining the poi (2,-3) and (5,6).

Soln: let required ratio = k:1 then coordinates of the poi of division are $\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$.

A/Q $\frac{6k-3}{k+1} = 0$

$6k-3=0 \Rightarrow 6k=3 \Rightarrow k=\frac{3}{6}=$

$\boxed{\text{required ratio} = 1:2}$ As

Q11. show (0,5), (0,-9) and (3,6) are collin

Soln. $x_1=0 \quad x_2=0 \quad x_3=3$
$y_1=5 \quad y_2=-9 \quad y_3=6$

Area $= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$= \frac{1}{2}|0(-9-6)+0(6-5)+3(5+9)|$

$= \frac{1}{2}|0+0+42|$

$= 21$

$\neq 0$

So, points are not collinear. As

Q12. Show that the points A(3,1) B(12,-2) a C(0,2) can not be vertices of a trian

Soln: $x_1=3 \quad x_2=12 \quad x_3=0$
$y_1=1 \quad y_2=-2 \quad y_3=2$

$$\text{Ar. of } \Delta = \frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2) \right|$$
$$= \frac{1}{2}\left[3(-2-2)+12(2-1)+0(1+2)\right]$$
$$= \frac{1}{2}\left[-12+12+0\right]$$
$$= \frac{1}{2} \times 0$$
$$= 0$$

Ar. of $\Delta = 0$ So, it is not a pointe of a.

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13. Find m if $(5,1)$ $(-2,-3)$ and $(8,2m)$ are collinear.

Soln: $x_1 = 5 \quad x_2 = -2 \quad x_3 = 8$

$y_1 = 1 \quad y_2 = -3 \quad y_3 = 2m$

For collinear points $A = 0$

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2) = 0$$
$$5(-3-2m)-2(2m-1)+8(1+3) = 0$$
$$-15-10m-4m+2+32 = 0$$
$$-14m+19 = 0$$
$$-14m = -19$$
$$\boxed{m = \frac{19}{14}}$$ Ans

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14. Find the coordinates of R on line joinin P(−1,3) and Q(2,5) such that $PR = \frac{3}{5}PQ$.

Soln: $PR : RQ = 3:2$

$m_1 = 3, \ m_2 = 2$

P $(-1,3)$ x_1, y_1 — R (x,y) — Q $(2,5)$ x_2, y_2

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2} \qquad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$x = \frac{3\times2+2\times-1}{3+2} \qquad y = \frac{3\times5+2\times3}{3+2}$$

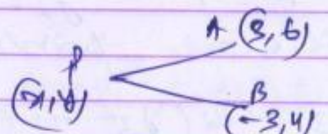
$$x = \frac{6-2}{5}, \quad y = \frac{15+6}{5}$$

$$x = \frac{4}{5}, \quad y = \frac{21}{5}$$

$$A = \left(\frac{4}{5}, \frac{21}{5}\right) \text{ Ans.}$$

Q15. Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Soln:



$$PA = PB \Rightarrow PA^2 = PB^2$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\cancel{x^2} + \cancel{9} - 6x + \cancel{y^2} + 36 - 12y = \cancel{x^2} + \cancel{9} + 6x + \cancel{y^2} + 16 - 8y$$

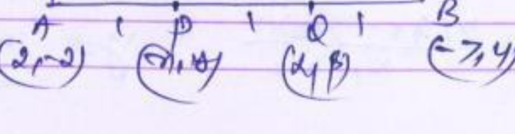
$$-12x - 4y = 16 - 36$$

$$-12x - 4y = -20$$

$$\boxed{3x + y = 5} \text{ Ans}$$

Q16. Find the coordinates of the points of trisection of the line joining the points $A(2, -2)$ and $B(-7, 4)$.

Soln:



$$x = \frac{1\times -7 + 2\times 2}{1+2}, \quad y = \frac{1\times 4 + 2\times -2}{1+2}$$

$$\boxed{P = (-1, 0)}$$

Now Q divides AB as 2:1 ratio

$$\alpha = \frac{2\times -7 + 1\times 2}{1+2}, \quad \beta = \frac{2\times 4 + 1\times -2}{2+1}$$

$$\alpha = -4, \quad \beta = 2$$

$$\boxed{Q = (-4, 2)} \text{ As.}$$

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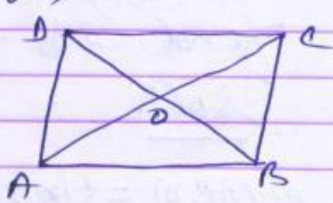
7. If the points A(6,1), B(8,2), C(9,4) and D(p,3) are the vertices of a ||gm taken in order find p.



coordinates of O on AC = coordinates of O on BD

$$\frac{6+9}{2}, \frac{1+4}{2} = \frac{8+p}{2}, \frac{2+3}{2}$$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}\right), \left(\frac{5}{2}\right)$$

$$\frac{8+p}{2} = \frac{15}{2} \quad \text{or} \quad \frac{5}{2} = \frac{5}{2}$$

$$p = 15 - 8$$

$$\boxed{p = 7} \text{ As}$$

Q/8. Find the area of a triangle whose vertices are $(1,-1)$, $(-4,6)$ and $(-3,-5)$.

Sol^n:

$$A = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$A = \frac{1}{2}\left|1(6+5)-4(-5+1)-3(-1-6)\right|$$

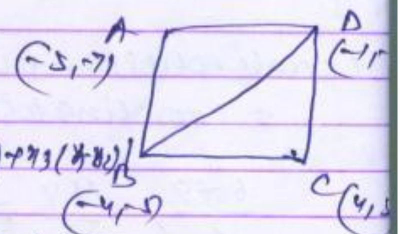
$$A = \frac{1}{2}\left|11+16+21\right|$$

$$A = \frac{48}{2}$$

$$\boxed{A = 24 \text{ sq. unit}} \text{ AS}$$

Q/9. If $A(-5,7)$, $B(-4,-5)$, $C(-1,-6)$ and $D(4,5)$ are vertices of quad. ABCD find its area.

Sol^n:



$$ar(ABD) = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|-5(-5-5)-4(5-7)+4(7+5)\right|$$

$$= \frac{1}{2}(50+8+48)$$

$$Ar = 53 \text{ sq. unit}$$

$$ar(BCD) = \frac{1}{2}\left|-4(-6-5)-1(5+5)+4(-5+6)\right|$$

$$= \frac{1}{2}\left|44-10+4\right|$$

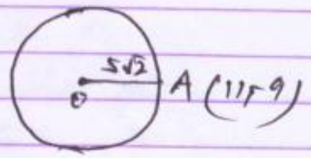
$$= 19 \text{ square unit}$$

$$ar(ABCD) = 53+19 = 72 \text{ sq. unit}$$

AS.

Q20. If the centre of circle is $(2a, a-7)$, find a if circle passing through $(11,-9)$ and has radius $5\sqrt{2}$ units.

Solⁿ:



$\therefore OA = 5\sqrt{2}$

$OA^2 = 50$

$(2a-11)^2 + (a-7+9)^2 = 50$

$4a^2 + 121 - 44a + a^2 + 4 + 4a = 50$

$5a^2 - 40a + 125 - 50 = 0$

$5a^2 - 40a + 75 = 0$

$a^2 - 8a + 15 = 0$

$a^2 - 5a - 3a + 15 = 0$

$a(a-5) - 3(a-5) = 0$

$(a-5)(a-3) = 0$

$a-5 = 0$ or $a-3 = 0$

$\boxed{a = 5 \text{ or } a = 3}$