BOOLEAN ALGEBRA

1 MARK QUESTIONS

1. Write the POS form of boolean function H, which is represented in a truth table as follows:

X	*Y*	*Z*	*H*
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

2. Write the SOP form of boolean function G, which is represented in truth table as follows:

P	*Q*	*R*	*G*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

3. Write the POS form of boolean function *H*, which is represented in a truth table as follows:

A	*B*	*C*	*H*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

4. Write the POS form a boolean function G, which is represented in a truth table as follows:

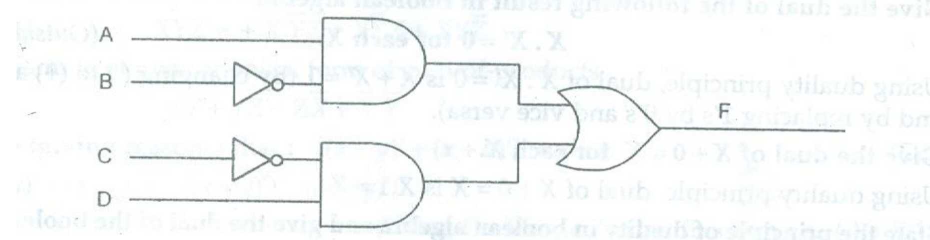
u	*v*	*w*	G
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

5. Draw a logic circuit diagram for the boolean expression: $\overline{X}.(\overline{Y} + Z)$
6. Draw a logic circuit diagram for the boolean expression: A.(B+C)
7. Draw a logic circuit diagram for the boolean expression: $\overline{A}$.(B+C)
8. Prove that X . (X + Y) = X by truth table method.
9. Find the complement of the following Boolean function:

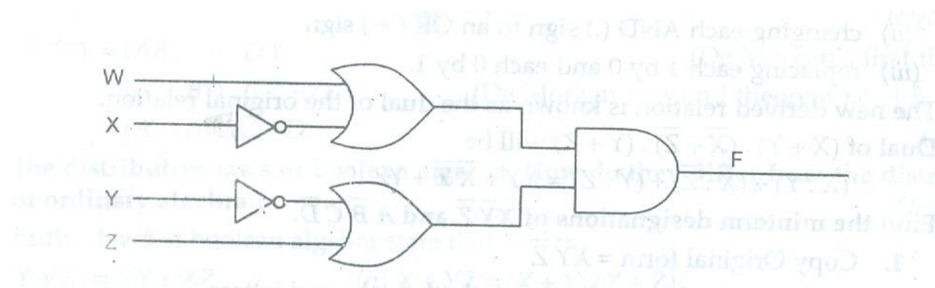
   $F_1 = AB' C'D'$
10. In the Boolean Algebra, verify using truth table that X + XY for each X, y in (0, 1).
11. In the Boolean Algebra, verify using truth table that (X + Y)' + X'Y' for each X' Y in (0, 1).
12. Give the dual of the following result in Boolean Algebra

   X . X' = for each X.
13. Define the followings:

   (a) Minterm (b) Maxterm (c) Canonical form
14. Interpret the following logic Circuit as Boolean expression:



15. Interpret the following Logic Circuit as Boolean Expression:



16. Write the dual of the Boolean expression A + B' .C.
17. Write the dual of the Boolean expression (B' + C) . A.
18. Represent the boolean expression X (Y' + Z) with help of NOR gates only.
19. State Demorgan's Laws:
20. Which gates are called Universal gates and why?

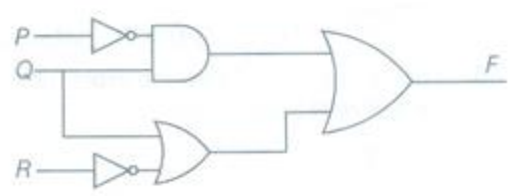
2 Marks Questions

1 State DeMorgan's laws. Verify one of the DeMorgan's laws using a truth table.

2 Draw a logic circuit for the following boolean expression.

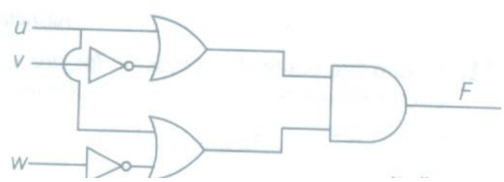
   A.B' +(C +B') .A'

3 Obtain the Boolean expression for the logic circuit shown below:

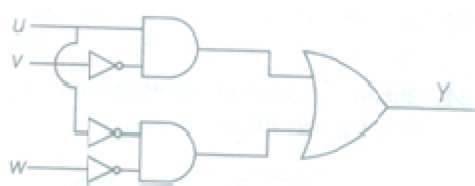


4 Verify the following using boolean expression using truth table:
(i) X + 0 = X
(ii) X + X' = 1

5 Write the equivalent Boolean expression for the following logic circuit:



6 Verify the following boolean expression using truth table:
(i) X . X' = 0 (ii) X + 1 = 1
7 Write the equivalent boolean expression for the following logic circuit:



8. Represent the Boolean expression X. Y' + Z with the help of NOR gates only.
9. Represent the Boolean expression (X + Y') .Z with the help of NAND gates only.

3 Marks Questions

1. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
   F(A, B, C, D) = ∑(1, 4, 5, 9, 11, 12, 13, 15)

2. Obtain a simplified form for the following boolean expression using Karnaugh's Map:
   F(P, Q, R, S) = ∑(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)

3. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
H(P, Q, R, S) = ∑(0, 1, 2, 3, 5, 7, 8, 9, 10, 14, 15)

4. Obtain the minimal form for the following boolean expression using Karnaugh's Map:
F(U, V, W, Z) = ∑(0, 1, 2, 3, 6, 7, 8, 9, 10, 13, 15)

5. Reduce the following boolean expression using K-map:
   F(P, Q, R, S) = ∑(1, 2, 3, 4, 5, 6, 7, 8, 10)

6. Reduce the following boolean expression using K-map:
   F(A, B, C, D) = ∑(2, 3, 4, 5, 6, 7, 8, 10, 11)

7. Reduce the following boolean expression using K-map:
   $F(A, B, C, D) = \sum(0, 1, 2, 4, 5, 6, 8, 10,)$

8. Reduce the following boolean expression using K-map:
   $F(P, Q, R, S) = \sum(0, 1, 2, 4, 5, 6, 8, 12)$

9. Reduce the following boolean expression using K-map:
   $F(A, B, C, D) = \sum(3, 4, 5, 6, 7, 13, 15)$

10. Reduce the following boolean expression using K-map:
    $F(u, v, w, z) = \sum(3, 5, 7, 10, 11, 13, 15)$

BOOLEAN ALGEBRA-SOLUTION

1 MARK QUESTIONS

1.

X	*Y*	*Z*	*H*	**Maxterm**
0	0	0	1	$X + Y + Z$
0	0	1	0	$X + Y + \overline{Z}$
0	1	0	1	$X + \overline{Y} + Z$
0	1	1	1	$X + \overline{Y} + \overline{Z}$
1	0	0	1	$\overline{X} + Y + Z$
1	0	1	0	$\overline{X} + Y + \overline{Z}$
1	1	0	0	$\overline{X} + \overline{Y} + Z$
1	1	1	1	$\overline{X} + \overline{Y} + \overline{Z}$

To get the POS form, we need to maxterms for all those input combinations that produce output as

0. Thus,

$$H(X, Y, Z) = (X + Y + \overline{Z}) .(\overline{X} + Y + \overline{Z}).(\overline{X} + \overline{Y} + Z)$$

2.

P	*Q*	*R*	*G*	**Minterm**
0	0	0	0	$\overline{P} . \overline{Q} . \overline{R}$
0	0	1	0	$\overline{P} . \overline{Q} . R$
0	1	0	1	$\overline{P} . Q . \overline{R}$
0	1	1	1	$\overline{P} . Q . R$
1	0	0	1	$P . \overline{Q} . \overline{R}$
1	0	1	0	$P . \overline{Q} . R$
1	1	0	1	$P . Q . \overline{R}$
1	1	1	1	$P . Q . R$

To get the SOP form, we need to sum minterms for all those combinations that produce output as 1. Thus,

$$G(P, Q, R) = (\overline{P} .Q .\overline{R}) + (\overline{P} .Q.R)+ (P .Q . \overline{R}) + (\overline{P} .Q. R)+ (\overline{P} .Q. R)$$

3.

A	***B***	***C***	***H***	**Maxterm**
0	0	0	0	$A + B + C$
0	0	1	1	$A + B + \overline{C}$
0	1	0	1	$A + \overline{B} + C$
0	1	1	1	$A + \overline{B} + \overline{C}$
1	0	0	1	$\overline{A} + B + C$
1	0	1	0	$\overline{A} + B + \overline{C}$
1	1	0	0	$\overline{A} + \overline{B} + C$
1	1	1	1	$\overline{A} + \overline{B} + \overline{C}$

To get the POS from, we need to product maxterms for all those input combinations that produce

output as 0. Thus,

$H(A, B, C) = (A + B + C).(\bar{A} + B + \bar{C}).(\bar{A} + \bar{B} + C)$.

4.

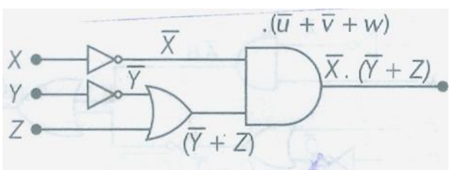
u	***v***	***w***	***G***	**Maxterm**
0	0	0	1	$u + v + w$
0	0	1	1	$u + v + \bar{w}$
0	1	0	0	$u + \bar{v} + w$
0	1	1	0	$u + v + \bar{w}$
1	0	0	1	$\bar{u} + v + w$
1	0	1	1	$\bar{u} + \underline{v} + \bar{w}$
1	1	0	0	$\bar{u} + \bar{v} + w$
1	1	1	1	$\bar{u} + \bar{v} + \bar{w}$

To get the POS form, we need to product maxterms for all those input combinations that produce output as

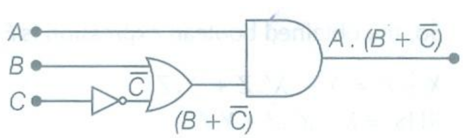
0. Thus,

$G(u, v, w) = (u + \bar{v} + w).(u + \bar{v} + \bar{w}).\ (u \bar{+} v \bar{+} w)\ ).(u \bar{+} v \bar{+} w)$

5.



6.



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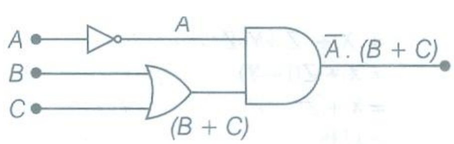


8.

X	Y	X + Y	X. (X + Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

From the above table it is obvious that X . (X + Y) = X because both the columns are identical.

9. (AB' +CD') = (AB')' . (C'D')' (De Morgan's first theorem)

= (A' + B'') . (C'' + D'') (De Morgan's second theorem i.e. $A.B = \bar{A} +$ B)

= (A’ + B) .(C + D) (X’’ = X)

10. As the expression X + XY is a two variable expression, so we require possible combinations of values of X, Y. Truth Table will be as follows:

X	Y	X + Y	X. (X + Y)
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Comparing the columns X + XY and X, we find, contents of both the columns are identical, hence verified.

11. As it is a 2 variable expression, truth table will be as follows:

X	Y	X + Y	(X + Y)’	X’	Y’	X’Y’
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Comparing the columns (X + Y)’and X’Y’, both of the columns are identical, hence verified.

12. Using duality principle, dual of X . X’ = 0 is X + X’ = 1 (By changing (.) to (+) and viceversa and by replacing 1’s by 0’s and vice versa).

13. (a) A Minterm is a product of all the literals (with or without the bar) within the logic system.
(b) A Maxterm is a sum of all the literals (with or without the bar) within the logic system.
(c) Aboolean expression composed entirely either of minterms or Maxterms is referred to as canonical expression.

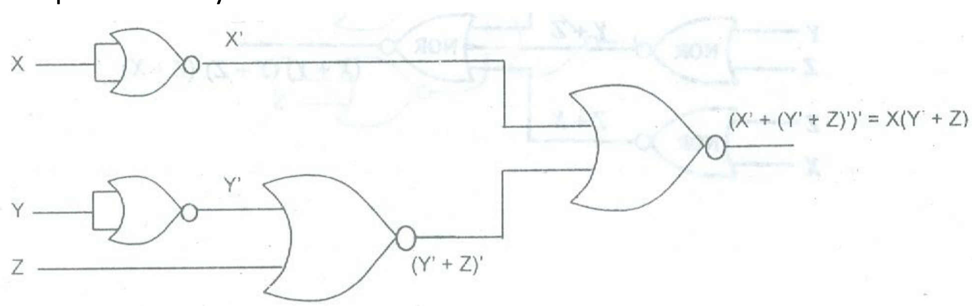
14. $F = A\overline{B} + \overline{C}D$

15. $F = (W + \overline{X})(\overline{Y} + Z).$

16. Dual of the Boolean expression *A + B’ .C is A .(B’ + C).*

17. Dual of the Boolean expression *(B’ + C) . A is (B’ .C) + A.*

18. The given expression may also be written as



19. De Morgan's first theorem. It states that X + Y = X . Y

De Morgan's second theorem. It states that X . Y = X + Y

20. NAND and NOR gates are less expensive and easier to design. Also, other switching functions and

(AND, OR) can easily be implemented using NAND/NOR gates. Thus, these (NAND/NOR) gates are also referred to *as Universal Gates.*

2 Marks Questions

1. DeMorgan's Laws:

It states that

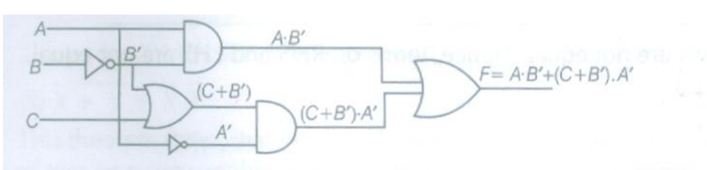
(i) $\overline{(A + B)} = \overline{A}.\overline{B}$ (ii) $\overline{(A. B)} = \overline{A} + \overline{B}$

Truth table for $\overline{A + B} = \overline{A}\,.\overline{B}$

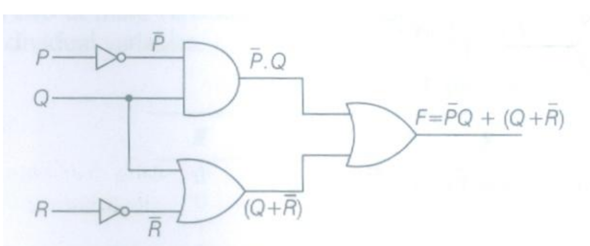
A	B	A+B	$\overline{(A+B)}$	$\overline{A}$	$\overline{B}$	$\overline{A}.\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Column 4 and Column 7 are equal, first law is proved.

2. *F*= A . B' + (C + B') . A'



3.



So, the obtained boolean expression is *F* = P'Q + (Q + R')

4. (i) X + 0 = X

X	0	X + 0
0	0	0
1	0	1

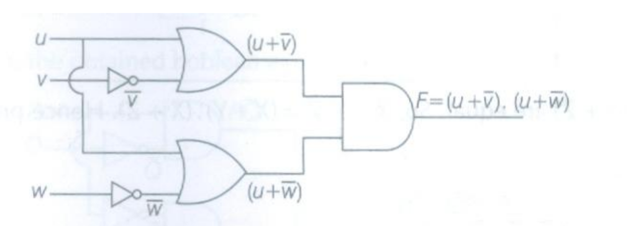
So, X + 0 + X

(ii) X + X' = 1

X	X'	X + X'
0	1	1
1	0	1

As X + X' = 1. Hence proved.

5.



So, the obtained boolean expression if *F*= $(u + \overline{v}).(u + \overline{w})$

6.

(i) X.X' = 0

X	X'	X.X'
0	1	0
1	0	0

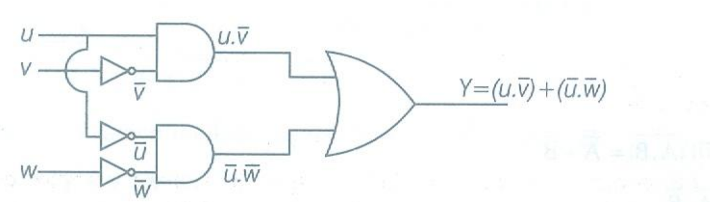
As X. X' = 0. Hence proved.

(ii) X + 1 = 1

X	1	X + 1
0	1	1
1	1	1

As X + 1 =1.Hence proved.

7.



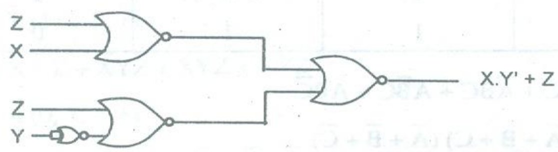
So, the obtained boolean expression is Y = $(u, \overline{v}) + (\overline{u} . \overline{w})$

law*)*

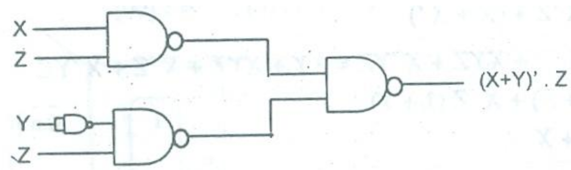
8. *X . Y' + Z*

$= Z + XY'$ $[X + Y = Y + X]$

$= (Z + X)(Z + Y')$ $[X + YZ = (X + Y)(X + Z)]$

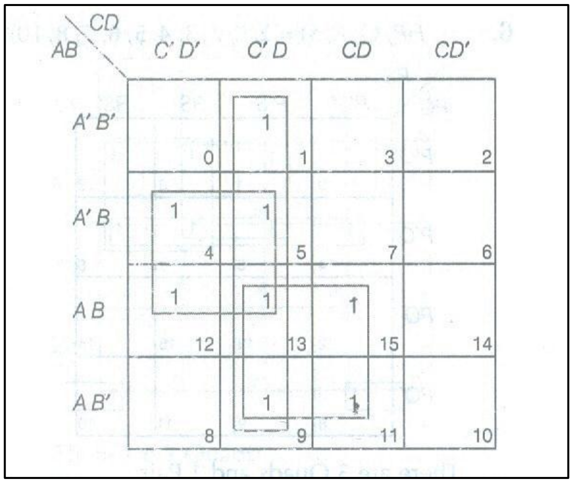


9. $(X + Y') . Z = X . Z + Y' . Z$



3 Marks Questions

1. $F(A, B, C, D) = \sum(1, 4, 5, 9, 11, 12, 13, 15)$



There are 3 Quads:

Quad 1 ($m_1 + m_5 + m_9 + m_{13}$) reduces to *C'D*

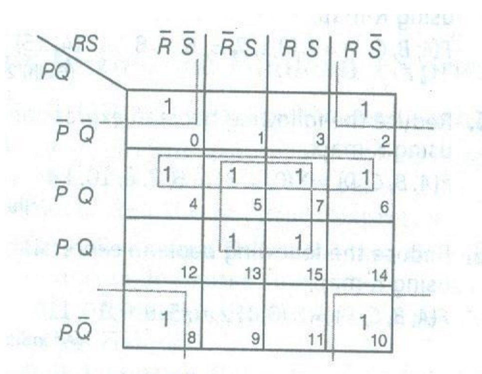
Quad 2 ($m_4 + m_5 + m_{12} + m_{13}$) reduces to *BC'*

Quad 3 ($m_9 + m_{11} + m_{13} + m_{15}$) reduces to *AD*

Hence, the final expression is:

F(A, B, C) = C'D + BC' + AD

2. $F(P, Q, R, S) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$



There are three Quads:

Quad 1 ($m_4 + m_5 + m_6 + m_7$) reduces to $\overline{P}Q$

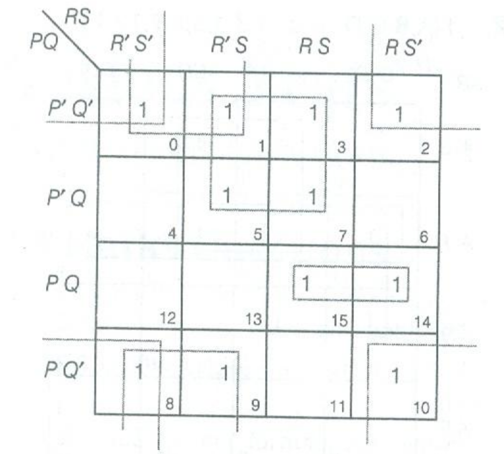
Quad 2 ($m_5 + m_7 + m_{13} + m_{15}$) reduces to QS

Quad 3 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{Q}\,\overline{S}$

Hence, the final expression is:

$F(P, Q, R, S) = \overline{P}Q + QS + \overline{Q}\,\overline{S}$

3. $H(P, Q, R, S) = \sum(0, 1, 2, 3, 5, 7, 8, 9, 10, 14, 15)$



There are three Quads and 1 Pair:

Quad 1 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{Q}\,\overline{S}$

Quad 2 ($m_1 + m_3 + m_5 + m_7$) reduces to $\overline{P}S$

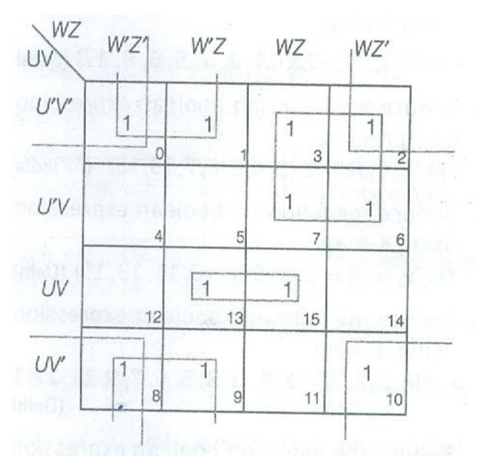
Quad 3 ($m_0 + m_1 + m_8 + m_9$) reduces to $\overline{Q}\,\overline{R}$

Pair 1 ($m_{14} + m_{15}$) reduces to PQR

Hence, the final expression is:

H(P,Q,R,S) = Q' S' + P' S + Q' R' + *PQR*

4. *F(U,V, W, Z)* = **∑**(0, 1, 2, 3, 6, 7, 8, 9, 10, 13, 15)



There are three Quads and 1 Pair:

Quad 1 ($m_0 + m_2 + m_8 + m_{10}$) reduces to $\overline{V}\,\overline{Z}$

Quad 2 ($m_0 + m_1 + m_8 + m_9$) reduces to $\overline{V}\,\overline{W}$

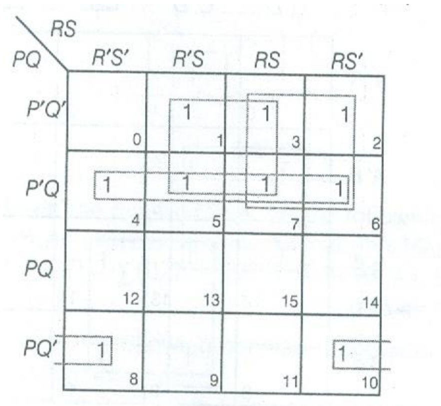
Quad 3 ($m_2 + m_3 + m_6 + m_7$) reduces to $\overline{U}\,W$

Pair 1 ($m_{13} + m_{15}$) reduces to *UVZ*

Hence, the final expression is:

F(U,V,W,Z)= V' Z' + V' W' + U' W +UVZ

5. *F(P, Q, R, S)* = **∑** (1, 2, 3, 4, 5, 6, 7, 8, 10)



There are three Quads and 1 Pair:

Quad 1 ($m_1 + m_3 + m_5 + m_7$) reduces to *P'S*

Quad 2 ($m_4 + m_5 + m_6 + m_7$) reduces to *P'Q*

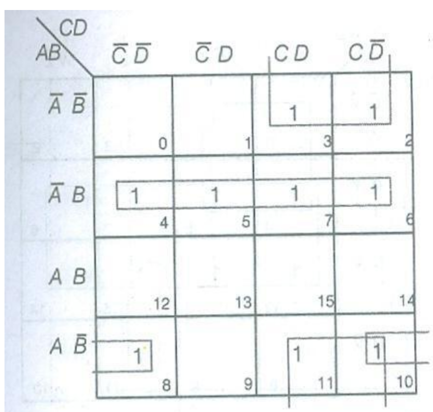
Quad 3 ($m_2 + m_3 + m_6 + m_7$) reduces to *P'R*

Pair 1 ($m_8 + m_{10}$) reduces to PQ'S'

Hence, the final expression is:

F(P,Q,R,S) = P'S + P'Q + P'R + PQ'S'

6. *F*(A, B, C, D) = Σ (1, 3, 4, 5, 6, 7, 12, 13)

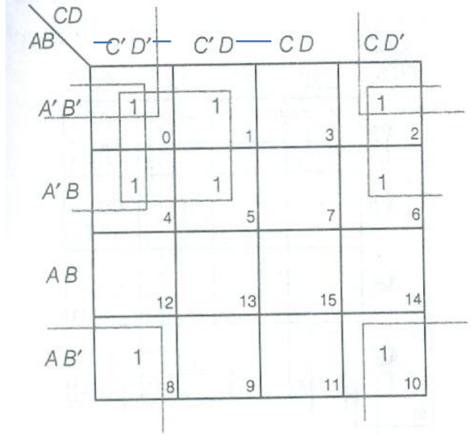


There are 2 Quads and 1 Pair:

Quad 1 ($m_4 + m_5 + m_6 + m_7$) reduces to $\overline{A}B$

Quad 2 ($m_2 + m_3 + m_{10} + m_{11}$) reduces to *B C*

Pair 1 ($m_8 + m_{10}$) reduces to *ABD*



Hence, the final expression is:

F(A, B, C,D) = AB +BC + ABD

7. *F*(A, B, C, D) = Σ (1, 2, 4, 5, 6, 8, 10)

There are 3 Quads:

Quad 1 ($m_0 + m_1 + m_4 + m_5$) reduces to *A'C'*

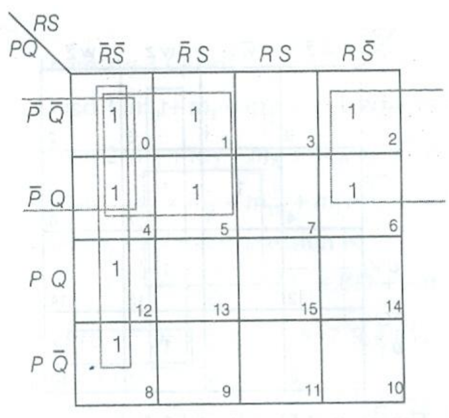
Quad 2 ($m_0 + m_2 + m_8 + m_{10}$) reduces to B' D'

Quad 3 ($m_0 + m_2 + m_4 + m_6$) reduces to *A' D'*

Hence, the final expression is:

F(A, B, C,D) = A'C' +B' D' + A' D'

8. *F(P, Q, R, S)* = ∑ (1, 2, 4, 5, 6, 8, 10, 12)



There are 3 Quads:

Quad 1 ($m_0 + m_4 + m_8 + m_{12}$) reduces $\bar{R}\,\bar{S}$

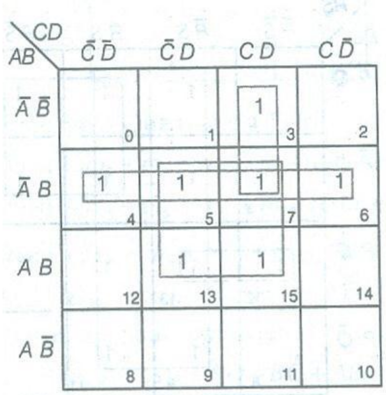
Quad 2 ($m_0 + m_1 + m_4 + m_5$) reduces $\bar{P}\,\bar{R}$

Quad 3 ($m_0 + m_2 + m_4 + m_6$) reduces $\bar{S}\,\bar{P}$

Hence, the final expression is:

F(P, Q, R, S) = $\bar{R}\bar{S} + \bar{P}\,\bar{R} + \bar{S}\,\bar{P}$

9. *F(A, B, C, D)* = ∑ (3, 4, 5, 6, 7, 13, 15)



There are 2 Quads and 1 Pair:

Quad 1 ($m_4 + m_5 + m_6 + m_7$) reduces to $\bar{A}B$

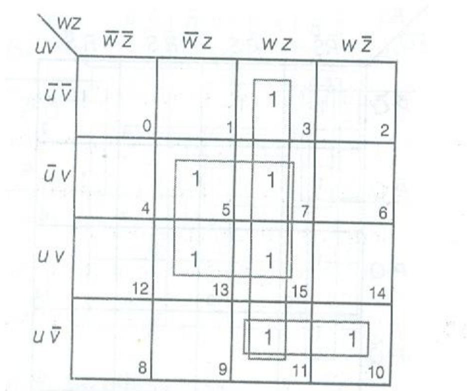
Quad 2 ($m_5 + m_7 + m_{13} + m_{15}$) reduces to BD

Pair 1 ($m_3 + m_7$) reduces to $\bar{A}CD$

Hence, the final expression is:

$F(A, B, C, D) = \bar{A}B + BD + \bar{A}CD$

10. $F(u, v, w, z) = \sum (3, 5, 7, 10, 11, 13, 15)$



There are 2 Quads and 1 Pair:

Quad 1 ($m_3 + m_7 + m_{11} + m_{15}$) reduces to wz

Quad 2 ($m_5 + m_7 + m_{13} + m_{15}$) reduces to vz

Pair 1 ($m_{10} + m_{11}$) reduces to $u\bar{v}w$

Hence, the final expression is:

$F(u, v, w, z) = wz + vz + u\bar{v}w$