

ARITHMETIC PROGRESSION

Q1. Write first 3 terms of series $a_n = n^2 + 1$

Soln:

$$a_n = n^2 + 1$$
$$a_1 = 1^2 + 1 = 2$$
$$a_2 = 2^2 + 1 = 5$$
$$a_3 = 3^2 + 1 = 10$$

Series: 2, 5, 10, - ...

Q2. Write an A.P. whose first term is 10 and c.d. is 3.

Ans:

$$10, 10+3, 10+3+3$$
$$10, 13, 16, - \dots$$

Q3. For A.P. $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ find first term and common difference.

Soln:

$$\boxed{a = \frac{1}{3}}, d = \frac{5}{3} - \frac{1}{3} = \boxed{\frac{4}{3}} \text{ Ans.}$$

Q4. Find 10th term of an A.P. 4, 9, 14, ... 25

Soln: $a = 4, d = 9 - 4 = 5$

$$t_n = a + (n-1)d$$

$$t_{10} = 4 + (10-1)5$$

$$t_{10} = 4 + 9 \times 5 = 4 + 45 = 49$$

$$\boxed{t_{10} = 49} \text{ Ans.}$$

Q5. Which term of A.P. -1, 3, 7, 11, ... is 95?

Soln: $a = -1, d = 7 - 3 = 4$

Let n th term be 95

$$t_n = 95$$

$$a + (n-1)d = 95$$

$$\begin{aligned}
 -1 + (n-1)4 &= 95 \\
 (n-1)4 &= 96 \\
 n-1 &= \frac{96}{4} = 24 \\
 n &= 24+1
 \end{aligned}$$

$$\boxed{n=25} \text{ Ans.}$$

Q6. How many terms are there in an A.P.
3, 6, 9, 12, ... 111 ?

Solⁿ: $a=3, d=6-3=3, t_n=111$

$$\begin{aligned}
 t_n &= 111 \\
 a + (n-1)d &= 111 \\
 3 + (n-1)3 &= 111 \Rightarrow 3(n-1) = 111-3 \\
 &\Rightarrow n-1 = \frac{108}{3} \\
 &\Rightarrow n = 36+1 \\
 &\Rightarrow \boxed{n=37} \text{ Ans.}
 \end{aligned}$$

Q7. Find the middle term of A.P. 6, 13, 20, ... 216

$$\begin{aligned}
 a &= 6, d = 13-6 = 7, t_n = 216 \\
 a + (n-1)d &= t_n \\
 6 + (n-1)7 &= 216 \\
 (n-1)7 &= 216-6 \Rightarrow 210 \\
 n-1 &= \frac{210}{7} \Rightarrow 30 \\
 n &= 30+1 \\
 &= 31
 \end{aligned}$$

Here n is odd so, Middle term = $\frac{(n+1)}{2}$ th term
M.T. = $\frac{31+1}{2} = \frac{32}{2} = 16$ th

$$\begin{aligned}
 t_{16} &= 6 + (16-1)7 \\
 &= 6 + 105 = \boxed{111 = t_{16}} \text{ Ans.}
 \end{aligned}$$

Q8. If 8th term of an AP is 31 and the 15th term is 16 more than the 11th term. find

Soln: Let a + 1st term
 d + C.d.

$$\text{A/Q } t_8 = 31 \quad \text{and } t_{15} = 16 + t_{11}$$

$$a + (8-1)d = 31$$

$$a + (15-1)d = 16 + a + t_{11}$$

$$a + 7d = 31$$

$$d + 14d = 16 + d + t_{11}$$

from (1)

$$a + 7 \times 4 = 31$$

$$14d - d = 16$$

$$a + 28 = 31$$

$$4d = 16$$

$$a = 3 \quad \text{--- (1)}$$

$$d = 4 \quad \text{--- (2)}$$

So, $[AP = 3, 7, 11, 15, \dots]$

Q9. Is 184 a term of AP: 3, 7, 11, ... ?

Let 184 is the n th term.

$$\therefore t_n = 184$$

$$a = 3, d = 4$$

$$a + (n-1)d = 184$$

$$3 + (n-1)4 = 184$$

$$(n-1)4 = 184 - 3 \Rightarrow 181$$

$$n-1 = \frac{181}{4}$$

$$n = \frac{181}{4} + 1 = \frac{185}{4} = 46 \frac{1}{4}$$

Here n is not a natural no.

So, 184 is not a term of given AP.

Q10. If 5 times the fifth term of an AP is equal to 8 times its 8th term. Show that its thirteenth (13th) term is 0.

solⁿ: Let $a =$ first term
 $d =$ common difference

$$\begin{aligned} \text{A/d} \quad 5\text{th} &= 8 \text{ to} \\ 5(a+4d) &= 8(a+7d) \\ 5a+20d &= 8a+56d \\ -3a &= 36d \\ 3a+36d &= 0 \\ 3(a+12d) &= 0 \\ a+12d &= 0 \\ \boxed{13} &= \text{Ans.} \end{aligned}$$

11. How many numbers of two digits are divisible by 7?

solⁿ: AP = 14, 21, 28, ... 98
 $d = 7, a = 14, t_n = 98$

$$\begin{aligned} a+(n-1)d &= t_n \\ 14+(n-1)7 &= 98 \\ (n-1)7 &= 98-14 = 84 \\ n-1 &= \frac{84}{7} \\ n &= 12+1 \\ \boxed{13} &= \text{Ans.} \end{aligned}$$

12. Two APs have same c.d. The first term of one of these is 3 and that of other is 8. What is the difference between their 10th terms.

Solⁿ: Let C.D. of two AP = d
and their n th terms are:

$$a_n = 8 + (n-1)d \text{ and } b_n = 8 + (n-1)d$$

$$a_n - b_n = 8 + (n-1)d - 8 - (n-1)d = 3 - 3 = 0$$

$$a_n - b_n = -5$$

$$\text{Hence } \boxed{a_{10} - b_{10} = -5} \text{ Ans}$$

Q13. The sum of 3 terms in AP is -3 and product is 0. Find no^s.

Solⁿ: Let $a-d$, a , $a+d$ are 3 terms

$$\text{A/Q } a-d + a + a+d = -3$$

$$3a = -3$$

$$a = -1$$

$$\text{and } (a-d) \cdot a \cdot (a+d) = 0$$

$$(a^2 - d^2) \cdot a = 0$$

$$(-1)^2 - d^2 = 0$$

$$1 - d^2 = 0$$

$$-d^2 = -1$$

$$d^2 = 1$$

$$d = \pm 1$$

$$\boxed{\text{Nos are } -1, -1, 2 \text{ or } 2, -1, -1} \text{ Ans}$$

Q14. If $2x$, $x+10$, $3x+2$ are in A.P. find

$$\text{Solⁿ: } x+10 - 2x = 3x+2 - x-10$$

$$-x+10 = 2x-8$$

$$-3x = -18 \Rightarrow x = \frac{18}{3} = 6$$

$$\boxed{x=6} \text{ Ans.}$$

Q15. Find the sum of 20 terms of AP
1, 4, 7, 10, ...

Soln: Here $a=1$, $d=4-1=3$, $n=20$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{20} = \frac{20}{2} [2 \times 1 + (20-1)3]$$

$$= 10 [2 + 57]$$

$$= 10 \times 59$$

$$\boxed{S_{20} = 590} \text{ Ans.}$$

Q16. If the n th term of an A.P. is $(2n+1)$, find the sum of first n terms of A.P.

Soln:

$$t_n = 2n+1$$

$$t_1 = 2 \times 1 + 1 = 3$$

$$t_2 = 2 \times 2 + 1 = 5$$

$$t_3 = 2 \times 3 + 1 = 7$$

$$a = 3, d = 5 - 3 = 2$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{n}{2} [2 \times 3 + (n-1)2]$$

$$= \frac{n}{2} [6 + 2n - 2]$$

$$= n \times (n+2) = \boxed{n^2 + 2n = S_n} \text{ Ans.}$$

Q17. Find the sum of first 30 terms
 an A.P. whose 2nd term is 2 and
 7th term is 22.

Solⁿ: $S_{30} = ?$, $t_2 = 2$ and $t_7 = 22$
 $a + d = 2$ ~~$a + 6d = 22$~~

⑩ ~~⊖~~ we get $5d = 20 \Rightarrow d = 4$
 put $d = 4$ in ⑩ ~~$a = 2 - 4 = -2$~~

$$S_{30} = \frac{30}{2} [2 \times -2 + (30-1) 4]$$

$$= 15 [-4 + 116]$$

$$= 15 \times 112$$

$$\boxed{S_{30} = 1680} \text{ Ans}$$

Q18. Find the sum of all natural nos
 between 200 and 1000 which is
 divisible by 5.

Solⁿ: AP: 252, 255, 258, ... 999
 $a = 250$, $d = 3$, $l = 999 = t_n$

$$a + (n-1)d = t_n$$

$$252 + (n-1)3 = 999$$

$$(n-1)3 = 999 - 252$$

$$n-1 = \frac{747}{3} = 249$$

$$n = 249 + 1$$

$$n = 250$$

$$S_n = \frac{n}{2} [a + l]$$

$$S_{250} = \frac{250}{2} [2.52 + 999]$$
$$= 125 \times 1001.5$$

$$S_{250} = 125187.5 \text{ Rs.}$$

Q19. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan.

Solⁿ: Here $a = 20$, $d = 15$

$$S_n = 3250$$

$$\frac{n}{2} [2 \times 20 + (n-1) \times 15] = 3250$$

$$\frac{n}{2} [40 + 15n - 15] = 3250$$

$$n(15n + 25) = 6500$$

$$15n^2 + 25n - 6500 = 0$$

$$3n^2 + 5n - 1300 = 0$$

$$(n-20)(3n+65) = 0$$

$$n = 20 \text{ or } n = -\frac{65}{3}$$

$$[n = 20]$$

Thus, the loan is cleared in 20 months.

Ans.

Q. Do. of $5n = 5n^2 + 3n$ find its nth term
soln:

$$5n = 5n^2 + 3n$$

$$5n = 5n^2 + 3n - [5(n-1)^2 + 3(n-1)]$$

$$5n = 5n^2 + 3n - [5n^2 - 7n + 2]$$

$$\boxed{5n = 10n - 2} \text{ Ans.}$$