TRIGONOMETRIC RATIOS

Q1. evaluate: $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

soln: $\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ As

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Q2. Prove that: $4(\sin^4 30 + \cos^4 60) - 3(\cos^2 45 - \sin^2 90) = 2$

LHS:

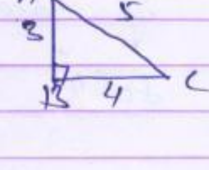
$$4 \times \left\{ \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^4 \right\} - 3\left\{ \left(\tfrac{1}{\sqrt{2}}\right)^2 - (1)^2 \right\}$$

$$= 4 \times \left(\tfrac{1}{16} + \tfrac{1}{16}\right) - 3\left(\tfrac{1}{2} - 1\right)$$

$$= \cancel{4} \times \frac{2}{\cancel{16}_{4_2}} + 3 \times \frac{1}{2}$$

$$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \text{ Proved}$$

Q3. If $\sin A = 3/5$ find $\cos A$ and $\tan A$



soln $AB^2 + BC^2 = AC^2$

$BC^2 = 5^2 - 3^2 = 25 - 9$

$BC^2 = 16$

$BC = 4$

$\cos A = \frac{4}{5}$ $\tan A = \frac{3}{4}$ $\cot A = \frac{4}{3}$, $\sec A = \frac{5}{4}$

$\operatorname{cosec} A = \frac{5}{3}$

Q4. If A, B, C are interior angles of a $\triangle ABC$ prove that $\tan \frac{B+C}{2} = \cot A/2$

soln: $A + B + C = 180$

$\frac{B}{2} + \frac{C}{2} = 90 - A/2$

$\tan \frac{B+C}{2} = \tan 90 - A/2$

$\tan \frac{(B+C)}{2} = \cot A/2$ Proved

Q5. If $\sin 5\theta = \cos 4\theta$ find θ.

Soln.

$$\sin 5\theta = \sin(90 - 4\theta)$$
$$5\theta = 90 - 4\theta$$
$$9\theta = 90$$
$$\boxed{\theta = 10^\circ} \text{ Ans}$$

Q6. If $\tan A = \cot B$ prove $A+B = 90^\circ$

Pf→ $\tan A = \tan(90-B)$

$$A = 90 - B$$
$$\boxed{A+B = 90^\circ}$$

Q7. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ find A.

Soln.

$$\operatorname{cosec}(90-4A) = \operatorname{cosec}(A-20^\circ)$$
$$90 - 4A = A - 20$$
$$90 + 20 = A + 4A$$
$$110 = 5A$$
$$A = \frac{110}{5}$$
$$\boxed{A = 22^\circ} \text{ Ans.}$$

Q8. Prove that $\tan 10 \tan 15 \tan 75^\circ \tan 80 = 1$

Pf→ L.H.S.

$\tan(90-80)\tan(90-75)\tan 75 \tan 80$

$= \cancel{\cot 80}\ \cancel{\cot 75}\ \cancel{\tan 75}\ \cancel{\tan 80}$

$= 1$

$=$ R.H.S

7. Prove that $\sin 35 \sin 55 - \cos 35 \cos 55 = 0$

LHS.

$\sin(90-55)\sin 55 - \cos(90-55)\cos 55$

$= \cos 55 \sin 55 - \sin 55 \cos 55$

$= 0$

$=$ R.H.S.

. If $\sin(A+B) = 1$, $\cos(A-B) = \frac{\sqrt{3}}{2}$ find A and B

Sol^n: $\sin(A+B) = \sin 90$

$A+B = 90$ — ①

$\cos(A-B) = \cos 30$

$A-B = 30$ — ②

① + ② we get $2A = 120$

$\boxed{A = 60}$

Put in ① $60 + B = 90$

$B = 90 - 60$

$\boxed{B = 30}$ Ans.

. Find θ, when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$.

Sol^n: $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

apply C & D rule

$\frac{\cos\theta - \cancel{\sin\theta} + \cos\theta + \cancel{\sin\theta}}{\cos\theta - \sin\theta - \cancel{\cos\theta} - \sin\theta} = \frac{1-\cancel{\sqrt{3}}+1+\cancel{\sqrt{3}}}{\cancel{1}-\sqrt{3}-\cancel{1}-\sqrt{3}}$

$\frac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\boxed{\theta = 30^\circ}$ Ans

Q12. If $x = 30°$ verify $\cos 3x = 4\cos^3 x - 3\cos x$

Sol^n: $\cos 3\times 30 = 4\times \cos^3 30 - 3\cos 30$

$\cos 90° = 4\times\left(\frac{\sqrt{3}}{2}\right)^3 - 3\times\frac{\sqrt{3}}{2}$

$0 = \cancel{4}\times\frac{3\sqrt{3}}{\cancel{8}\,2} - \frac{3\sqrt{3}}{2}$

$0 = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$0 = 0$ proved

LHS = RHS

Q13. Find θ if $\sqrt{3}\tan 2\theta - 3 = 0$

Sol^n: $\sqrt{3}\tan 2\theta = 3$

$\tan 2\theta = \sqrt{3}$

$\tan 2\theta = \tan 60$

$2\theta = 60$

$\boxed{\theta = 30°}$ As

Q14. Prove that $\frac{\cos 30 + \sin 60}{1 + \cos 60 + \sin 30} = \frac{\sqrt{3}}{2}$

LHS. $\frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}}$

$= \frac{\frac{\cancel{2}\sqrt{3}}{\cancel{2}}}{2}$

$= \frac{\sqrt{3}}{2}$

$=$ RHS proved

Q15. If θ is an acute angle and $\sin\theta = \cos\theta$ find the value of $2\tan^2\theta + \sin^2\theta - 1$.

Sol: $\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = 45^\circ$

$2 \times \tan^2 45 + \sin^2 45 - 1$

$= 2 \times 1^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 + \frac{1}{2} - 1$

$= 1 + \frac{1}{2}$

$\boxed{= \frac{3}{2}}$ Ans.

Q16. Write the max. and min. value of $\sin\theta$

Ans -1 and 1.

Q17. What is the max. value of $\frac{1}{\sec\theta}$ and $\frac{1}{\operatorname{cosec}\theta}$

Ans. 1 and 1

Q18 Prove: $(1-\sin^2\theta)\sec^2\theta = 1$

LHS $\cos^2\theta \cdot \sec^2\theta$

$= \cos^2\theta \cdot \frac{1}{\cos^2\theta}$

$= 1$

$=$ R.H.S.

Q19. $\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta + \cot\theta$ Prove it

LHS. $\frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$$= \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

$= R.H.S$

Q.20 Prove: $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

LHS.

$$\frac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta-1)}$$

$$= \tan\theta \left[\frac{1-2(1-\cos^2\theta)}{2\cos^2\theta-1}\right]$$

$$= \tan\theta \left[\frac{1-2+2\cos^2\theta}{2\cos^2\theta-1}\right]$$

$$= \tan\theta \left[\cancel{\frac{2\cos^2\theta-1}{2\cos^2\theta-1}}\right]$$

$$= \tan\theta$$

$= RHS.$